USING ICT IN TEACHING A SPECIFIC MATHEMATICS CONCEPT: GRAPHS OF LOGARITHMIC FUNCTIONS

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This paper describes the use of the free software, to teach graphs of logarithmic functions at an Ethiopian College of Teacher Education. Data comprised two video-recorded lessons and interview data provided by a mathematics teacher educator, and three primary school mathematics pre-service teachers who were in the class of learners. Pre-service teachers using Microsoft Mathematics readily understood and described properties of logarithmic functions as the bases varied. The study highlights the importance of illustrating the use of particular software to teach specific mathematics concepts.

INTRODUCTION

The use of Information Communication Technology (ICT) in teaching can lead to significant positive pedagogical outcomes (e.g., Goos, Galbraith, Renshaw & Geiger, 2003; Pierce & Stacey, 2010)). Such findings have motivated universities, colleges, and school teachers to integrate ICT into teaching to achieve better learning outcomes. ICT can support constructivist pedagogies, whereby students use technology to explore and to reach an understanding of concepts (Chee, Horani, & Daniel, 2005). As a result, integration of ICT in teaching is a key component of educational reform agenda to enhance the quality of education across the world. For example, countries in Africa strongly endorse and support ICT as an essential component of innovative student-centred pedagogy (e.g., Hennessy, Harrison, & Wamakote, 2010).

The current Ethiopian school curriculum and education system have been characterised as low in quality (e.g., Desta, Chalchisa, Mulat, Berihun, & Tesera, 2009). There is, therefore, considerable support throughout the system, including in the higher education sector, to improve quality. Efforts have been made to encourage teachers at all levels to integrate ICT in their teaching. These include national initiatives that encourage teacher educators to use ICT in their teaching as a means to improve the quality and equity of education, particularly for science and mathematics teaching (Ministry of Education, 2010). This study aligns with that effort in exploring how an Ethiopian mathematics teacher educator used *Microsoft Mathematics* (MSM) to teach the graphs of logarithmic functions.

ICT IN LEARNING MATHEMATICS

Although the integration of ICT in teaching has generic aspects as described, for example by Koehler and Mishra (2005), there is also a need to consider the use of technology in particular subjects such as mathematics and indeed in relation to specific

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mathematics content (Holmes, 2009). In addition, the affordances of specific technological tools influence possible teaching approaches and appropriate pedagogies (Kennewell, 2001).

There is a great deal of mathematics specific software available (Hohenwarter, Hohenwarter, & Lavicza, 2009) and ICT-based learning environments can provide opportunities for active learning and enhanced student engagement (Chee, Horani, & Daniel, 2005). For instance, simulations and animations enable students to vary a selection of input parameters, observe how each affects the system under study, and interpret the output results through an active process of hypothesis-making, and ideas testing. They can explore combinations of factors and observe their effects on the evolution of the system under study. Mathematics specific software includes Geogebra, MSM, Maxima, STELLA, and spread sheets. MSM is free and can help students to achieve an understanding of a range of mathematical concepts. It can help students to visualise the effects of changed parameters. The MSM interface allows for solving problems with minimal syntax instruction (Nord & Nord, 2011) and facilitates animation. The use of the 'Animate' command found within MSM can possibly aid discovery-style lessons (Morrison, Tversky, & Betrancourt, 2000). It offers, for example, visualisation of shapes of graphs of families of logarithmic functions by learners input of bases, b, between $b > 1$, and $0 < b < 1$.

In Ethiopia, financial constraints mean that freely available software is preferable. MSM was selected for this study because it fitted this criterion and also had capabilities thought to be useful in enhancing the teaching of mathematical ideas such as logarithmic functions.

Logarithmic functions

Functions, including logarithmic, form a major part of school mathematics but are challenging to teach (Makgakga & Sepeng, 2013). Kenney and Kastberg (2013) found that students struggled greatly with the concept of logarithmic functions and sketching their corresponding graphs, and with the processes needed for working with logarithmic equations. Superficially, the graphs of all logarithmic function can easily be overgeneralised as similar shapes regardless of varied bases, b (e.g., Chua & Wood, 2005). In this regard Goos, Galbraith, Renshaw and Geiger (2003) indicated that the introduction of technology resources into mathematics classrooms promises to create opportunities for enhancing students' learning through active engagement. In addition, a study by Abu-Naja (2008) showed that students learn more effectively the characteristic properties of families of functions using technology than without using any ICT. This study, therefore, focuses on the use a particular software resource, MSM, to teach a specific mathematical topic, namely graphs of logarithmic functions.

METHOD

The Study

The research site was the department of mathematics in an Ethiopian college of teacher education. Primary mathematics pre-service teachers (PSTs) are required to attend basic mathematics and professional courses in a 3-year program. Mathematics courses include Fundamental Concepts of Algebra, Plane Geometry, Basic Mathematics I and II, and Introduction to Calculus. The professional courses included Methods of Teaching Mathematics.

Participants

Participants were PSTs finishing their first year in the program. They were enrolled in Basic Mathematics II, the content of which includes graphs of logarithmic functions. Most were aged between 18 and 24 years. All 29 (18 males and 11 females) participated in the observation part of the study. A mathematics teacher educator who had participated in a professional learning program process for a total of 3 months, aimed at encouraging the use of ICT in initial teacher education also participated.

Procedure

Two video-recorded lessons totalling 2 hours and involving the use of MSM to teach logarithmic functions were taught by the teacher educator. Once the PSTs were familiar with the menus and toolbars of the software, they learned how to graph logarithmic functions. They were then asked to work in groups of three or four to illustrate properties of graphs of logarithmic functions. The questions shown in Figure 1 were provided to guide their work.

1) Sketch the graphs of the following logarithmic functions:
   a. $f(x) = \log_2 x$
   b. $f(x) = \log_5 x$
   c. $f(x) = \log_{\frac{7}{2}} x$
   d. $f(x) = \log_{\frac{2}{3}} x$
   e. $f(x) = \log_{\frac{1}{2}} x$
2) Describe the shapes of the graphs when $b > 1$, and $0 < b < 1$
3) Describe properties of the graphs listed in question 1
   a. Common properties
   b. Describe the graphs when $x > 1$, and $0 < x < 1$, $x = 1$

Figure 1: Questions explored using MSM

Following the two lessons, semi-structured, audio-recorded, individual interviews were conducted with three PSTs (two males and one female) and the teacher educator. The teacher educator was asked for his views of the lessons he taught with MSM and about his previous teaching of graphs of logarithmic functions (such as how the lesson engaged learners?). PSTs were asked for their opinions of the MSM integrated lessons (e.g., How engaging the lesson was? What aspects of the lesson helped them to learn?).

Data analysis

Interview data from PSTs' and the teacher educator were analysed to identify themes (Creswell, 2009) relevant to using MSM to teach graphs logarithmic functions. The video-recorded lessons were analysed by watching and taking notes (Stigler & Hiebert, 1997) emphasising those parts relevant to the questions shown in Figure 1. Consistent with the advice of Barron and Engle (2007), the analysis emphasised aspects of ICT use known to be relevant, such as how the students interacted with MSM, specifically their use of the tools it provided, and how they worked to make sense of their graphs.

RESULTS

The teacher educator's previous approach to teaching logarithmic functions

The teacher educator described two methods he had previously used to teach sketching graphs of logarithmic functions. The first involved taking a simple logarithmic statement, switching it around to the corresponding exponential statement, and then figuring out the *x*-value needed for that exponent (*y*-value). The second, the T-chart method, is carried out by taking powers of the base of the function as *x*-values and finding the corresponding *y*-values. The teacher educator identified this method as preferred because it requires learners to know the procedures for finding the values of logarithmic functions. For example, to draw the graph of $\log_2(x)$, PSTs first list some values of *x* and *y* on the T-chart and then sketch the graph by connecting points as indicated in Figure 2. He acknowledged that this method is challenging for comparing multiple graphs on the same axes. For example, it is difficult to exactly identify which graph approach the y axis when $x > 1$, and $0 < x < 1$. The remaining results are presented in three sections corresponding to the questions in Figure 1.

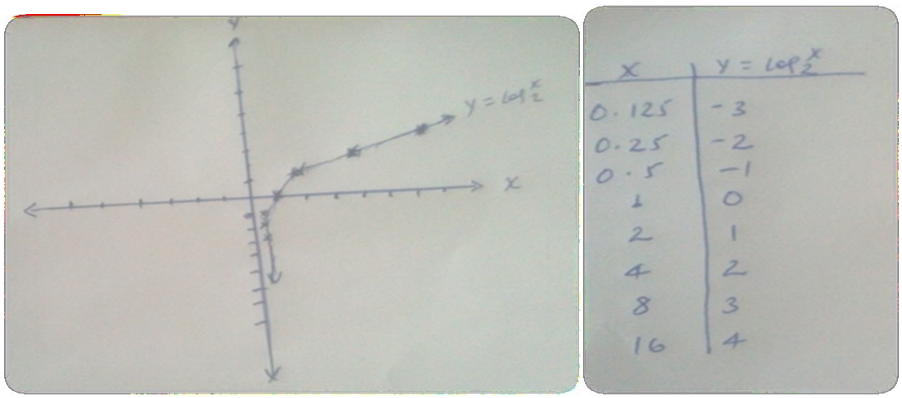

Figure 2: T-chart and graph for $f(x) = \log_2(x)$

Graphing logarithmic functions

The teacher educator began by presenting the definition of the logarithmic function, $y = \log_b(x)$, where *b* is any number such that $b > 0$, $b \neq 1$ and $x > 0$. Using MSM, each group of PSTs was able to draw multiple graphs of logarithmic functions easily, with distinct colours, and on the same axes, as illustrated in Figure 3. They were required to write the equation in the "writing box" and click on the icon 'graph' to find the graph of the corresponding equation, and appeared to enjoy sketching the graphs.

During the interview a PST pointed to the effect of using the software on learners' engagement while admitting incomplete understanding of what was happening. She said:

> The software helped me to easily sketch each graph on the same *x*-*y* axis with distinct colours; however, I don't know clearly how it happens. (PST 1)

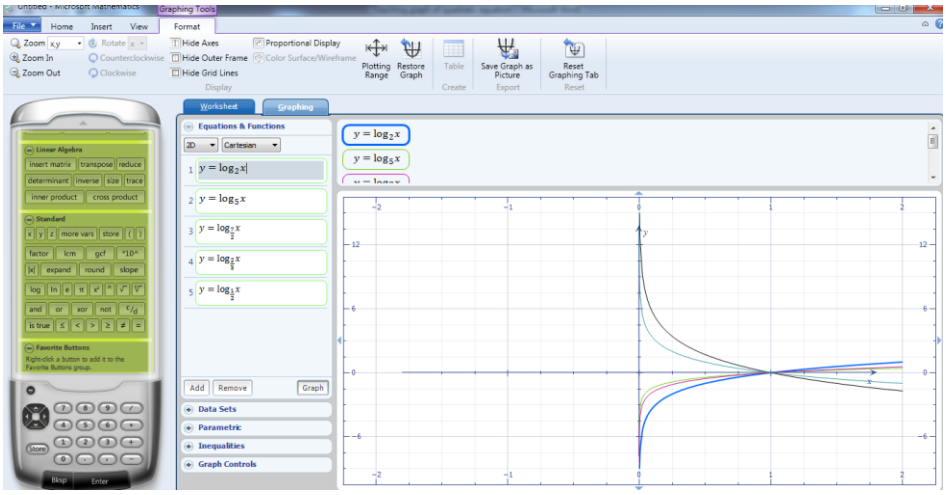

Figure 3: Graphs of some logarithmic functions created in MSM

Describing shapes of the graphs when $b > 1$, and $0 < b < 1$

Using MSM, the PSTs were able to describe the shapes of the graphs with a general equation $f(x) = \log_b(x)$ without sketching multiple graphs but rather by changing the value of b between $b > 1$, and $0 < b < 1$ using the "Animate" feature of MSM to generate a movie of different graphs as b changed. Alternatively, b could be directly controlled by inputting a value. Using the animate icon, PSTs observed and described the shapes of logarithmic functions for values of b between 0 and 2. In the video-recorded lesson, they appeared to recognise and appreciate the shape change when b hurdles 1. During the interview, a PST indicated his interest in these animations.

> I liked the role of "animate" to clearly see the shape of the graphs of multiple logarithmic functions as the base b varies without sketching samples of multiple graphs. (PST 3)

PST1 indicated the impression as b hurdles 1 as:

> By using animate function I was able to understand the graph approached positive y axis as $b < 1$, whereas, it approached negative y-axis as $b > 1$. (PST1)

Describing the common properties of the graphs of logarithmic functions

With MSM the groups of PSTs readily identified that all logarithmic functions have the same general shape, with their graphs varying depending on the base and coefficients in the equation. During the lessons, PSTs were pointing to the graphs made using MSM to identify and describe the common properties (For example, the fact that all have a vertical asymptote at $x = 0$, and cross the x-axis at $x = 1$) of the logarithmic function in each category, $b > 1$, and $0 < b < 1$. When interviewed a PST described the usefulness of MSM as follows:

> I liked the software which helped to graph all logarithmic functions on the same *x*-*y* axis with different colours. This helped me to list and understands the common properties of logarithmic function as the base, *b* varies. (PST 2)

Describe shapes of the graphs when $x > 1$, $0 < x < 1$, $x = 1$

Most groups of PSTs described the shapes of the graphs by observing the sketched graphs. However, one group was observed trying to identify the properties of the graphs through 'Trace' function of MSM. The trace function varies the values of x continuously and y for a given base b, as x moves through a specified range of values. In this case PSTs identified the values of y as x moved between $x > 1$, $0 < x < 1$.

Summary of the teacher educator and PSTs' reflections on the lessons

PSTs expressed a range of perspectives on the use of MSM in learning graphs of logarithmic functions. One of the PSTs had mixed feelings about using MSM, expressing a preference to use both. Although she recognised the significance of technology, she tended to believe that graphs of logarithmic function should be first taught without using any ICT then later by MSM. This was the same PST who had admitted being unsure of how MSM produced the graphs. Another explained the advantage of MSM comparing with his previous lessons. He said:

> At the first glance, the graph of the logarithmic function can easily be mistaken for that of the square root function when sketching manually. Both the square root and logarithmic functions have a domain limited to x values greater than 0. However, the logarithmic function has a vertical asymptote descending towards negative ∞ as x approaches 0, whereas the square root reaches a minimum y value of 0. This difference was clearly demonstrated by using MSM. (PST 3)

Another PST indicated that MSM helped to externalise his reasoning, work at his own pace, and manage the complexity of the task scaffolding pen-and-paper skills. He said:

> MSM complements my learning of graphs of logarithmic function by helping to visualise, understand, and animate to identify their properties. ... I liked the process as I was engaged and discussed with peers throughout the process and it was a different approach. (PST 2)

The teacher educator described the role of MSM as follows:

> The software was vital and complements PSTs' ability to discuss the problem by engaging PSTs in a small group guided by me. The discussion within their small groups was thought provoking as they were engaged through manipulating the computer. I liked MSM as it complements my efforts by helping PSTs to visualise graphs of logarithmic functions as well as provoked active engagement of PSTs.

DISCUSSION AND CONCLUSION

The study illustrated the use of specific software to teach a specific mathematics topic for understanding. MSM provided a variety of utilities that were able to engage PSTs to relearn, and reorganise their knowledge of graphs of logarithmic functions. Although the interviewed PSTs recognised benefits of technology, one of them believed that the topic should be taught with traditional methods before being explored

using technology. Chee et al. (2005) claimed that such a preference can be due to the difficulty teachers have in adopting appropriate pedagogies for particular software. Given the inexperience of teaching with technology of the teacher educator in this study inexpert pedagogy may underpin this PST's opinion as well as her difficulty in understanding exactly what was going on.

PSTs readily used MSM to visualise graphs and identify their properties. The animate facility allowed them to display the graphs as desired based on changing parameters, and helped to facilitate discovery-style lessons (Morrison et al., 2000). These software capabilities were particularly important for the chosen mathematics content because difficulties had been identified in relation to students' ability to distinguish the graphs of different logarithmic functions (Chua & Wood, 2005). In addition, MSM supported the PSTs to describe the graphs of logarithmic functions and appeared to support their understanding of the topic. The usefulness of MSM in learning about the graphs of logarithmic function appeared due to its ability to:

- Facilitate the learning processes through making it easier to produce graphs of logarithmic functions on the same axes accurately,
- Make the lesson more engaging through enabling the tasks based on trial, improvement and experimentation,
- Help PSTs to notice the effects of altering particular parameters (in this case the base of a logarithmic function) on the properties of the function's graph, and
- Foster PSTs' peer exchange through providing support for exploration and consequent sharing of discoveries.

Although focussed on the use of specific software (MSM) to teach a specific mathematical concept (graphs of logarithmic functions), the study suggests that software with similar capabilities (graphing and animation) could be useful for other functions in which visualisation and change in graph characteristics across the function domain are important features. It has also demonstrated the potential of freely available software to help teachers in developing countries and other contexts in which resources are limited.

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